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Translated from the Russian by
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Translator's note:

Symbols:

| | |
|---|---|
| н | means the starting or initial point |
| к | " " finishing or the end point |
| л | should be read as an English l (*ℓ*) |

The above symbols appear in the text and diagrams of the translation.

# THE CALCULATION OF A THERMAL BOUNDARY LAYER IN THE FLOW OF A COMPRESSIBLE GAS

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A semi-empirical approximation method which permits calculation with adequate accuracy of the laminar, transient and turbulent regions of a thermal boundary layer during flow of compressible gas over its surface, is examined in this article.

The solution of many technical problems is often associated with the necessity of calculating the heat exchange of a surface with the compressed gas flowing over it. The boundary layer appearing under such circumstances and dependent on the flow conditions and the nature of velocity distribution along the surface, may either be laminar, transient or turbulent on the overall surface; or it may be laminar, then transient, and then turbulent on parts of the surface.

In this article a semi-empirical approximation method is proposed, which permits calculation with sufficient accuracy of all three thermal regions of a boundary layer appearing during the flow of compressible gas over its surface. The essence of the method is as follows.

Let us examine a plane flow of a compressible gas. It is known that, in this case, the integral relationship of energy in Dorodnitsin variables appears as:

$$\frac{d\delta_T^{**}}{d\xi} + \frac{U'_{0\xi}}{U_0} = \frac{T_0^*}{T_w}\frac{\mathrm{Nu}_x}{\mathrm{Pr}\mathrm{Re}_x}. \qquad (1)$$

Here

$$\delta_T^{**} = \int_0^\delta \frac{\rho}{\rho_0^*}\frac{u}{U_0}\left(1 - \frac{t^*}{t_0^*}\right)dy; \qquad (2)$$

$$U'_{0\xi} = \frac{dU_0}{d\xi};\ U'_0 = \frac{dU_0}{dx};\ \xi = \int_0^x \frac{p}{p_0}\,dx;$$

$$\mathrm{Nu}_x = \frac{\alpha x}{\lambda^*};\ \mathrm{Re}_x = \frac{U_0 x}{\nu^*};\ \mathrm{Pr} = \frac{\nu^*}{a^*}; \qquad (3)$$

$$t^* = T^* - T_w;\ t_0^* = T_0^* - T_w \qquad (4)$$

(sign * pertains to the parameters of drag).

It may be written as

$$\frac{\partial}{\partial x}(1 - \alpha_0^2)^{\frac{k}{k-1}}\frac{\partial}{\partial\xi} + \frac{\partial\eta}{\partial\xi}\frac{\partial}{\partial\eta}, \qquad (5)$$

where

$$\sigma_0 = \frac{U_0}{\sqrt{2Ic_pT_0}};\ \eta = \int_0^\delta \frac{\rho}{\rho_0^*}\,dy. \qquad (6)$$

Using relationship (5), equation (1) may be reduced to appear as:

$$\frac{d\delta_T^{**}}{dx} + \frac{U'_0}{U_0}\delta_T^{**} = \frac{T_0^*}{T_w}(1 - \alpha_0^2)^{\frac{k}{k-1}}\frac{\mathrm{Nu}_x}{\mathrm{Pr}\mathrm{Re}_x}. \qquad (7)$$

Introducing the parameters:

$$f_T = \frac{U'_0}{U_0}\delta_T^{**}G_T, \qquad (8)$$

$$\chi = \frac{T_0^*}{T_w}(1 - \alpha_0^2)^{\frac{k}{k-1}}\frac{\mathrm{Nu}_x}{\mathrm{Pr}\mathrm{Re}_x}G_T \qquad (9)$$

let us assume that they, changing along the streamlined surface, uniquely determine all characteristics of a thermal boundary layer. Let us also assume that in expression (8) the effect of a longitudinal gradient of pressure is characterized by complex $\frac{U_0' \delta_T^{**}}{U_0}$, and $G_T$ is not dependent on $\frac{du}{dx}$ and only determines the effect of Reynolds number. In this case, value $G_T$ will be the same for the flow around the profile as well as along the lamina. We determine function $G_T$ by data on the heat exchange of the lamina.

By examining the test data (Fig 1) it is apparent that during deviation of the physical constants toward the parameters of drag*, the formulas for calculating heat exchange retain the same form as in the case of an incompressible flow.

For the transient region of a boundary layer the influence of compressibility reflects on the coordinates of the start and the finish of transition $(x_н, x_к)$; and the development of the transition process, after its appearance, occurs in such a manner that the lines $\mathrm{Nu}_x = N(\mathrm{Re}_x)$ remain parallel to each other in the whole investigated diapason of the change in the number M.

Evidently these curves can be approximated by series:

$$\mathrm{Nu}_x = B\,\mathrm{Re}_x^n, \qquad (10)$$

where values B and n are different, but are invariable for each regime of flow in the boundary layer. For a transient region coefficient B, as the tests show,

*During the calculations all physical constants deviate toward the temperature of drag.

is a variable value, changing with the change in the value of Reynolds number, which corresponds with the initial point of $Re_{xH}$ transition. This value is determined by the initial turbulence of the flow, by the value of the temperature factor and the number M., i.e., value B retains a constant value within the limits of each concrete test, but can change with the change of the initial and operational conditions of the process.

Substituting expression (10) into the integral energy relationship for a lamina:

$$\frac{d\delta_T^{**}}{dx} = \frac{T_0^*}{T_w}(1-\alpha_0^2)^{\frac{k}{k-1}} \frac{\mathrm{Nu}_x}{\mathrm{PrRe}_x} \qquad (11)$$

and using formula (9), we can obtain the following expression for $G_T$:

$$G_T = (m+1)\left[\frac{\mathrm{Pr}}{(m+1)B}\right]^{m+1} \frac{\chi}{\left[\frac{T_0^*}{T_w}(1-\alpha_0^2)^{\frac{k}{k-1}}\right]^{m+1}} \mathrm{Re}_T^{**m}, \qquad (12)$$

where $m = \frac{1-n}{n}$.

We determine function $\chi$ in such a manner that the following may be acceptable for the lamina:

$$\chi\left[\frac{T_0^*}{T_w}(1-\alpha_0^2)^{\frac{k}{k-1}}\right]^{-(m+1)} = 1. \qquad (13)$$

Then the formula for $G_T$ is obtained:

$$G_T = A\mathrm{Re}_T^{**m}, \qquad (14)$$

which is completely analogous to the formula for an incompressible flow /1/.

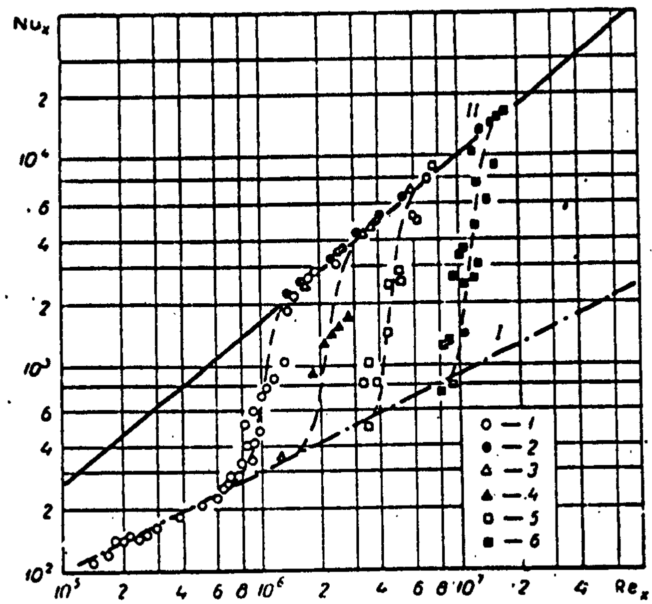



Fig.1. Dependence of $Nu_x = N\ (Re_x)$ at different values of the number M: 1-0.24; 2-0.44; 3-0.52; 4-0.73; 5-1.07; 6-1.43; I-$Nu_x = 0.297\ Re_x^{1/2}$- laminar boundary layer; II- $Nu_x = 0.0255\ Re_x^{4/5}$- turbulent; $T_w/T_0^* \sim 1$.

By introducing relationships (8), (9), and (14) into equation (7) and performing simple calculations, it is possible to reduce it to the following form:

$$\frac{df_T}{dx} = [(m+1)\chi - 2f_T]\,\frac{U_0'}{U_0} - \frac{U_0''}{U_0'}\,f_T. \qquad (15)$$

For incompressible flow it was experimentally shown /2/ that the function, placed in brackets

$$F_T = (m+1)\chi - 2f_T \qquad (16)$$

may, with sufficient accuracy, be expressed by relationship:

$$F_T = a - 2f_T. \tag{17}$$

With this it was found that value $\alpha$ is not dependent on the gradient of pressure, but is determined only by the regime of flow in the boundary layer.

Let us assume that an analogous relationship may also be written for a compressed flow. If $\alpha$ is not dependent on the longitudinal gradient of the pressure, then it evidently should have the same value for the flow along the profile, as well as for the flow along the lamina. Then, using expression (13), $\alpha$ may be determined by formula

$$a = (m+1)\left[\frac{T_0^*}{T_w}(1-\alpha_\infty^2)^{\frac{k}{k-1}}\right]^{m+1}, \tag{18}$$

where $\alpha_\infty$ is value $\alpha_0$ for the undisturbed flow.

When examining the flow, characterized by conditions $T_w/T^*_0 = \theta =$ const, it is evident that for each regime of the flow $\alpha$ will be a constant value and dependent only on the regime of the flow.

In this case equation (15) is easily integrated and allows to determine $\delta^{**}_T$ for the thermal boundary layer:

$$\delta_T^{**} = \left(\frac{a}{A}\right)^{\frac{1}{m+1}} \frac{(\nu_0^*)^{\frac{m}{m+1}}}{U_0} \left\{ \int_{x_H}^{x} U_0 dx + \nu_0^* \, (\mathrm{Re}_{TH}^{**})^{m+1} \, \frac{A_n}{a} \right\}. \tag{19}$$

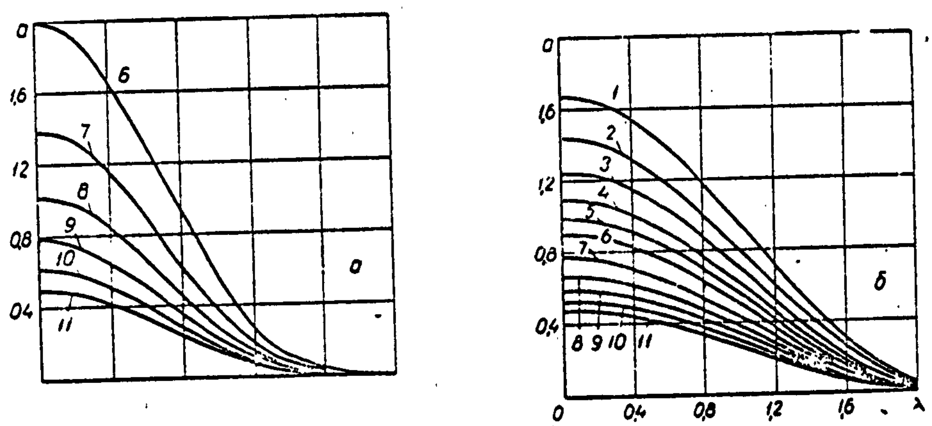
Here $A_1$ corresponds to value of coefficient A, in formula (14) for the laminar boundary layer if the calculation is carried out for a transient region, and corresponds to value A for a transient region if the calculation is carried out for a turbulent section of the boundary layer. Accordingly, $Re^{**}_{T_H} = \frac{U_0 \delta^{**}_{T_H}}{\nu_0}$ is determined for coordinates of the start of transition from calculation of the laminar section in the first case, and for coordinates of the finish of transition from calculation of the transient region in the second case. For convenience and speed up of the calculation, values $\alpha$ for each regime of flow in the boundary layer may be calculated by formula (18) in the required diapason of changes of paramters and be presented in the form of curves /Fig 2 /.

Using formulas (19), (9), (16) - (18) an expression may be obtained for calculation of local values of the heat exchange coefficient

$$Nu_x = \\ = \left(\frac{m+1}{A}\right)^{\frac{1}{m+1}} \frac{Pr}{1+m} \left[\frac{1-\alpha_\infty^2}{1-\alpha_0^2}\right]^{\frac{k}{k-1}} Re_x \left\{\int_{x_H}^{x} \frac{U_0 dx}{\nu_0} + (Re^{**}_{T_H})^{m+1} \frac{A_n}{a}\right\}^{-\frac{m}{m+1}}. \quad (20)$$

By comparing (20) with a corresponding formula for an incompressed flow(4) it is discovered that, during deviation of all physical constants toward the temperature of drag of flow $T_0^*$, a formal analogy exists between the appearance of the formula for calculation of the heat exchange intensity in a compressed flow $Nu_x$, and an incompressed flow $Nu'_x$. By comparing these formulas a relationship is easily obtained

$$\mathrm{Nu}_x = \mathrm{Nu}'_x \left[\frac{1-\alpha_\infty^2}{1-\alpha_0^2}\right]^{\frac{k}{k-1}} = \mathrm{Nu}'_x \left[\frac{1-\frac{k-1}{2}\lambda_\infty^2}{1-\frac{k-1}{2}\lambda_0^2}\right]^{\frac{k}{k-1}}. \tag{21}$$



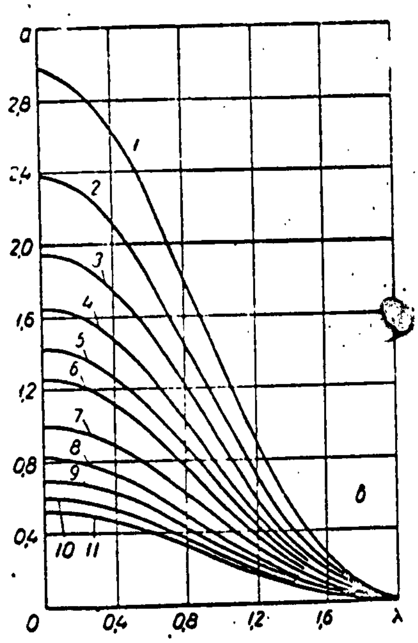




Fig 2. Dependence of $\alpha = \alpha(\lambda)$ for a laminar region of the boundary layer (a); for a transient region (b) and for a turbulent region (c) at different $T_w/T_o^*$ :

1—0,5; 2—0,6; 3—0,7; 4—0,8; 5—0,9; 6—1,0; 7—1,2; 8—1,4; 9—1,6; 10—1,8; 11—2,0

It is seen from this formula that in the instance of a flow over the lamina with velocity on the outer boundary layer equaling the velocity of the inflowing stream and corresponding to $\alpha_o = \alpha_\infty$ or $\lambda_o = \lambda_\infty$, then with

the above described representation of the formulas, the following relationship is valid:

$$Nu_x = Nu_x'. \qquad (22)$$

As was already mentioned, this formal relationship corresponds well with the test data.

During calculation the coordinate points of the start of transition $X_h$ may be determined by Dorodnitsin - Loitsinskii method /3/. For an approximate determination of the transient region, the following simple consideration may be recommended. Processing of the tests shown in Fig. 1 and analogous tests of other authors indicated that paramter $r_x$, characterizing the relationship between the coordinates of the finish $X_k$ and the start of transition $X_h$, do not depend on the number M. This characteristic of parameter $r_x$ permits determination of its value along empirical curves obtained for an incompressed flow and presented in article /4/.

## DESIGNATIONS

$\rho_o^*$ is the density corresponding to parameters of drag; $U_o$. $T_o$ is the velocity and the temperature outside the boundary layer; $T_w$ is the temperature of the wall.

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